# The Asymmetric Effects of Monetary Policy on Stock Market 

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#### Abstract

This paper investigates the asymmetric effects of monetary policy on the U.S. stock market across different monetary policy regimes and stock market phases. In particular, it uses a Markov-switching dynamic factor model to generate a new composite measure that represents the overall stock market movements, and dates the turning points of each bear market and bull market. A time-varying parameter analysis, which is undertaken in the framework of a state space model and estimated via Kalman Filter, is then used to study the contemporaneous and lead-lag effects of monetary policy on stock returns. The results provide evidence that major changes in monetary regimes and shifts in stock market conditions shape the time-varying relationship between monetary policy and stock returns. It is observed that the monetary policy of changing monetary aggregates is less effective in bear markets than bull markets, but changes in federal funds rate can be more effective in bear markets. The results also indicate that increases in monetary aggregates or reductions in the federal funds rate have positive contemporary effects on stock performance only during the periods in which they are used as the monetary policy intermediate target by the Federal Reserve.


Key words: Monetary policy; Stock market; Markov switching; Dynamic factor model; Time-varying parameter model;

JEL Classification: G1; E44; E52

## 1. Introduction

The Federal Reserve has two ultimate objectives for its monetary policy: to support maximum sustainable output and employment, and to maintain stable price level. These two goals are explicitly announced in the 1977 amendment to the Federal Reserve Act. It is stated by mounting literatures on the transmission of monetary policy that the Federal Reserve affects real economy through the financial markets and especially the stock market. For instance, as Bernanke and Kuttner (2005) state, the effects of monetary policy on macroeconomic objectives are at best indirect and lagged, and the most direct and immediate influence of monetary policy is on the stock market. Many other studies also support the view that monetary policy has an instantaneous and significant impact on stock market (see, for example, Thorbecke, 1997; Patelis, 1997; Lastrapes 1998; Rigobon and Sack, 2004; Farka 2009, among others). Strength or weakness of the stock market can have a substantial impact on real activities such as consumption through the wealth effect and investment through the credit channel. Many believe that in the context of monetary policy management, the Federal Reserve must view its macroeconomic objectives and stock market sustainability as complementary and consistent goals, to be pursued within an integral policy framework. The commonly accepted wisdom is that expansionary monetary policy measures should have a positive effect on the stock performance.

Given the fact that monetary policy has significant influence on stock market, several cross-section studies have sought to investigate if monetary policy has asymmetric impacts on stock performance according to different firm characteristics such as its size and capital intensity. For example, Ehrmann and Fratzscher (2004) reached the conclusion that capital-intensive firms and financial-constraint firms are more strongly affected by monetary policy.

Some time-series studies (Durham 2001, 2003) showed that the relationship between monetary policy and stock market return is historically unstable and time-varying. However, there is not much done in the literature analyzing how and why the relationship varies over time. Is it possible that the time-varying response of stock return to monetary policy depends on drastic changes in monetary regimes or on the
phases of the stock market being in a bull or bear market?
The aim of this paper is to explore whether the effects of monetary policy on stocks are asymmetric over time depending on the phases of the stock market and the monetary policy regimes from 1970s to present. This topic has gained popularity in the current scenario of expansionary monetary policy and historically high stock price level in the U.S. Understanding the responsiveness of stock market to changes in monetary policy shed light on the transmission mechanism of monetary policy, since stock market performance plays an important role on real activities through many channels.

Investigating the impact of monetary policy across different stock market phases and monetary policy regimes naturally requires identifying the beginning and end of these phases and regimes. The periods of monetary policy regimes can be defined using the dates on which monetary policy targets changed, which is well-documented in the Federal Reserve's history. Yet, agreement on the dates of stock market turning points between bull and bear market regimes is far from unanimous. Moreover, there is no commonly accepted formal definition of bear and bull markets in academic literatures. In the U.S., the National Bureau of Economic Research (NBER) provides business cycle dates that are regarded as official. This dating is obtained by examining the comovement in the switch of several major economic variables.

This paper uses the NBER's principle together with Chauvet (1998) classification method to define the bull and bear markets by employing a Markov-switching dynamic factor model to date their turning points. The framework is cast in a state space model, and estimated via Kalman Filter (1960) and Hamilton Filter (1989). The dynamic factor model captures the clustering of shifts between upward and downward tendency of a variety of popular stock indices. The Markov-switching feature reflects the asymmetry of stock movements in terms of growth rate and volatility, and is able to statistically identify the date of turning points through the smoothed probabilities.

The results show that the model successfully captures all bear markets and bull markets in the sample. Moreover, the model also produces a new composite index that represents the stock market price movements more precisely and broadly. The new composite measure has advantages over existing stock indices, given that they are
criticized for their limitation on the coverage of certain types of stocks and stock exchanges. The Markov-switching dynamic factor model also calculates the average durations of bear and bull markets, and the probability of bear and bull market at every time point. These results can be instrumental in assisting investors and policy makers to understand in which state the stock market is and where the stock market will move towards.

In the next step, this paper uses the proposed new stock market movement index into a time-varying parameter model to explore the dynamic interrelationship between monetary policy and stock performance across different monetary policy regimes and stock market phases. Monetary policy is represented not only by short-term policy interest rate and but also by monetary aggregates to reflect the fact that these two variables have been used as the monetary targets in the Federal Reserve's history. The lead-lag relationship and contemporaneous relationship are analyzed in two separate time-varying parameter models, which are represented in the state space models, and estimated through the Kalman Filter and maximum likelihood estimation method. To the best of my knowledge, this article is the first to study this topic in the framework of Markov-switching dynamic factor model and time-varying parameter model. It can unveil features of their relationship that have not been captured previously.

The results show that major shifts in monetary policy can substantially impact the dynamic effects on stock return through changes in the monetary policy target. The contemporary signaling effect of federal funds rate changes impact the stock market only during periods in which the federal funds rate is used as monetary target by the Federal Reserve. This is also the case for monetary aggregates. That is, monetary aggregates affects stock market positively only during periods in which they are used as monetary targets in 1970s and 1980s. The findings also indicate that a positive predictive relationship between money supply and stock market occurs during the periods of strong economic growth, but not during the periods of economic recession or slow recovery.

This paper provides evidence of the asymmetric response of stock return to monetary policy during bear and bull markets. In fact, there is a sharp drop in the
correlation between monetary aggregate and stock returns in every bear market, indicating that the influence of expansionary monetary policy through increases in money supply is much weaker in a bear market, and can even have a negative effect on the stock market. However, an expansionary monetary policy through reduction in short-term policy interest rate is considerably effective in improving stock market returns.

The remainder of the paper is organized as follows. The next section discusses the studies conducted in the past literature. Section 3 describes the theoretic framework of the relationship between monetary policy and stock movements. The data are described in the fourth section. Section 5 illustrates the Markov-switching dynamic factor model and time-varying parameter model, which are the empirical models applied in this study. Section 6 presents the empirical results. This paper is concluded in the seventh section with some discussion of additional issues. Econometric estimation procedures are discussed in the Appendix.

## 2. Literature Review

### 2.1 Literatures on the U.S. stock market regimes

The fundamental understanding of a bull market is a period of substantial and continuous increase of stock prices, and a bear market is a period of substantial and continuous reduction in stock prices. Stock market commentators often define a bull market as a $20 \%$ or $25 \%$ stock price rise, and a bear market as a $20 \%$ or $25 \%$ stock price decline. Some financial analysts identify the beginning of a bear market when the 50-day moving average line crosses the 200-day moving average line from the below, and holds above. However, in the academic area, the finance and economics literatures have no commonly accepted definition of bull market and bear market. Several studies provided their own definitions of bull and bear markets, such as Chauvet and Potter (2000), Pagan and Sossounov (2003), and Chen (2007). For example, Chen (2007) used a simple Markov-switching model on stock returns to estimate the probabilities of bear market and bull market, and it found that the correlation between the bull market probability and the bull market binary variable constructed by using $20 \%$ cutoff line is round 0.7 .

### 2.2 Literatures on the U.S. monetary policy regimes

According to Meulendyke (2003) and Mishkin (2006), the Federal Reserve's monetary policy experienced substantial changes over the past four decades. In 1970, Arthur Burns was appointed chairman of Board of Governors of the Federal Reserve, and the Federal Reserve started to use monetary aggregates as intermediate target and federal funds rate as operating target to fight inflation, which was caused by the procyclical monetary policy. The Federal Open Market Committee (FOMC) selected growth rate for monetary aggregate, and chose a federal funds rate that would achieve that desired monetary aggregate growth rate. However, this monetary target policy was unsuccessful in controlling inflation, due to the fact that monetary aggregate target and federal funds rate may conflict with each other. The federal funds rate targeting led to a procyclical monetary policy which raised inflation pressure during the periods of economic expansion in the early 1970s. The economic contraction started from the middle of 1970s was associated with federal funds rate reduction and monetary aggregate growth sharp drop, which in turn made the economic condition worse. Combined with other inflation factors such as oil and agriculture products supply decrease, 1970s was mainly featured by stagflation.

In October 1979, Paul Volcker became chairman of Board of Governors of the Federal Reserve. The Federal Reserve's monetary policy has shifted into a new regime in 1980s. The main goal in this era is to change interest rate to fight serious inflation. The operating target was switched from federal funds rate into nonborrowed reserve and borrowed reserve sequentially. Monetary aggregate still served as the intermediate monetary target. A predetermined target path for nonborrowed reserve and borrowed reserve was based on the objective for the monetary aggregate. The federal funds rate was largely raised in early 1980s and the inflation was successfully controlled. However, this anti-inflation monetary strategy missed most monetary aggregate targets, indicating that monetary aggregate was deemphasized as the target.

When Alan Greenspan was elected as Federal Reserve's chairman in 1987, the Federal Reserve announced that it would no longer use monetary aggregate as its target. In 2000, legislation amending the Federal Reserve Act officially ceased to require the

Federal Reserve to report monetary aggregate target to Congress. Abandoning monetary aggregates as the guide for its monetary policy, the Federal Reserve has restarted to target federal funds rate since early 1990s. Periods in 1990s and 2000s were featured by the clear monetary policy goal in terms of macroeconomic variables, clear operating target which is federal funds rate, without an explicit intermediate target. This strategy is called "just do it" approach by Mishkin (2006). By actively and timely changing federal funds rate, the Federal Reserve tried to keep the economy and financial market on track. Ben Bernanke began his tenure in early 2006. The same monetary strategy continued until 2007, when a new and more complicated problem came up. Since 2008, a sufficient injection of bank reserves has brought the federal funds rate fundamentally close to zero, so that the zero lower bound rules out further policy interest rate reduction. The Federal Reserve has to seek alternative nontraditional monetary policy tools to improve the condition of financial market and promote the growth of economy, which are known as quantitative easing and forward guidance.

### 2.3 Literatures on general responsiveness of stock to monetary policy

The responsiveness of stock movements to monetary policy has been a matter of increased concern since 1980s. There is a body of literature investigating this issue. For most of these studies, monetary policy is divided into two main streams: changing the money supply and changing the policy interest rates.

The effects of expansionary monetary policy, such as increasing money supply and reducing policy interest rates, on the stock return are claimed to be positive in these empirical researches. Thorbecke (1997) employed a monthly VAR model for the period from 1967 to 1990 to analyze the link and used the federal funds rate to measure monetary policy. He found that the response of stock returns to a negative one standard deviation shock to the federal funds rate is $0.8 \%$. This empirical finding that a positive relationship between the expansionary monetary policy of reducing policy interest rate and stock return has been confirmed by Patelis (1997), Lastrapes (1998) and many others.

In a more recent study, Rigobon and Sack (2004) used the policy shocks that take place on certain dates such as the days of FOMC to examine this topic, and documented a positive linkage between expansionary monetary policy and stock movements. In a
similar vein, Bernanke and Kuttner (2005) took a more traditional event-study approach, while controlling directly for certain kinds of information jointly influencing monetary policy and stock return. They applied ordinary least squares regressions in an event study, and found that an unexpected 25 basis points decrease in the federal funds target rate is associated with a one percent increase in the stock prices.

But there is not yet a consensus on this conclusion, as several articles provide counter examples on the direction of effects. Cornell (1983) found the link between money supply announcement and asset prices can be either positive or negative, depending on the underlying assumption and hypothesis. He discussed three hypotheses (expected inflation hypothesis, Keynesian hypothesis, and real activity hypothesis) suggested in the previous literature as well as the risk premium hypothesis that he proposed. These results were consistent with those of other studies which have analyzed the relationship between monetary policy and the stock return. Lee (1997), for example, applied rolling regressions to measure the relationship between short-term interest rate and stock prices, which is measured by the S\&P 500 index, indicating an unstable linkage. Another effort along these lines is that of Garg (2008), who conducted empirical research about the effects of changes in federal fund rate on stock prices in different sectors. His work showed that stock prices and interest rate move in the same direction, indicating an expansionary monetary policy of reducing policy interest rates may deteriorate the stock performance. He also gave theoretical explanation for this seemingly surprise result.

There is some dissent on the response of stock market to the monetary policy among the existing literature. The direction of the reaction is impossible to determine ahead. Possible explanations for this dissent are provided in the theoretical framework section of this paper.

## 2. 4 Literatures on the asymmetric effects of monetary policy on stock return

Chen (2007) studied the monetary policy's asymmetric effects on stock returns in different stock market conditions, and found that monetary policy has a larger effect in less booming stock markets and stagnant stock markets. His finding indicated that a contracting monetary policy is more likely to cause a weak stock market. Jansen and Tsai
(2010) investigated the asymmetric impact of monetary policy on stock return in bull and bear market during the time period from 1994 to 2005, and showed that the monetary policy shocks in bear market is large, negative, and statistically significant. Kurov (2010) analyzed the stock returns on FOMC announcement days, and found that monetary policy shocks have strong influence on market participants' sentiment, and this impact is stronger in a bear market.

Jensen, Mercer and Johnson (1996) suggested that monetary policy regime affects investors' required return. They found that stock return is higher in tight monetary policy regime than expansionary monetary policy regime. Kual (1987) showed that the relationship among monetary policy, inflation, and stock return can be either positive or negative depending on whether monetary policy is pro-cyclical or counter-cyclical. Du (2006) supported this conclusion and found that changes in money supply and its consequential inflation can have different effects on stock returns during different monetary policy regimes. The results showed that there was a positive relationship among money supply, inflation and stock return during the period of pro-cyclical monetary policy regime, and this relationship became negative during the period of counter-cyclical monetary policy regime. Laopodis (2013) examined the dynamic relationship between monetary policy and stock market during the three distinct monetary policy regimes of Burns, Volcker and Greenspan since 1970s. It found there was a very weak relationship between monetary policy action via federal funds rate and stock return in 1990s. His paper provides evidence for asymmetric effects of monetary policy on stock in different monetary regimes and stock market conditions.

## 3. Theoretical Framework

### 3.1 Theoretical background of stock price valuation

Recall that the objective of this paper is to investigate the effects of monetary policy on stock price movements. To do so it is necessary to have a solid understanding of stock price valuation. The most popular theory for the stock valuation is the present value model or discounted cash flow model. This model is well explained by Crowder (2006) and Ioannidis Kontonikas (2008), among many studies. The intrinsic stock price $P_{t}$ is
valued as the present value of future expected dividends cash flows $D_{t+j}$ of the company and terminal stock price at the last period of holding horizon. Under the assumption of constant discounting rate, the present value model is expressed as follows,

$$
P_{t}=E_{t}\left[\sum_{j=1}^{N}\left(\frac{1}{1+R_{t}}\right)^{j} D_{t+j}\right]+E_{t}\left[\left(\frac{1}{1+R_{t}}\right)^{N} P_{t+N}\right]
$$

where $E_{t}$ is the conditional expectation operator based on the information available up to time $t, N$ is the number of investor's holding period, $R_{t}$ is the rate of return to discount the future values. As the stock holding periods $N$ increases to infinity, the second term on the right hand side of the equation vanishes to zero.

$$
\lim _{N \rightarrow \infty} E_{t}\left[\left(\frac{1}{1+R_{t}}\right)^{N} P_{t+N}\right]=0
$$

Therefore, the stock price valuation model can be described as follows

$$
P_{t}=E_{t}\left[\sum_{j=1}^{N}\left(\frac{1}{1+R_{t}}\right)^{j} D_{t+j}\right]
$$

According to the above theory, the intrinsic stock price is simultaneously determined by two parts: future cash flows and the discounting rate. Therefore, monetary policy can affect stock price through both future cash flows and discounting rate, which is linked to interest rate.

### 3.2 Theoretical background of the effects of monetary policy on stock price

The Federal Reserve has several monetary tools available, such as open market operations, discount loans, and required reserves. It also has the ability to set discount rate and federal funds rate target to affect the financial markets and real economic activities. It is widely accepted that all the monetary policy measures can be summarized into two major channels: changes in money supply and changes in short-term interest rate. These two measures are correlated most of the time, in that a rise of money supply in terms of bank reserves will put downward pressure on the short-term interest rate which clears the reserve market. In other words, an increase in money supply will generate a drop in interest rate. The only exception arise in the case of current zero lower bound interest rate, which already rules out further policy interest rate reduction. Only in this scenario can we separate the movements of money supply and interest rate. Given the main objective of this paper is to provide explanations to the anomalous results of QE1 and QE2 in case of the zero lower bound interest rate, it is appropriate to examine
the effect of change in money supply and change in interest rate separately.
It is commonly believed that expansionary monetary policy, considered as a rise in money supply or a reduction in short-term policy interest rate, can drive up the stock price by increasing the future cash flow and decreasing discounting rate. However, the actual mechanism behind is much more complicated. The impacts of expansionary monetary policy on stock market can be either positive or negative. In addition, the effects through these two channels can reinforce or offset each other.

In general, the response of stock prices to the expansionary monetary policy of reducing interest rate is positive. That is why there exists a long tradition for the Federal Reserve to drop short-term policy interest rates in an attempt to promote the stock market condition. The detailed reasons for the positive linkage are presented as follows. First, it is obvious that a lower interest rate indicates a lower discounting factor, implying a higher present value of future cash flows and hence a higher stock price, given that the future cash flows are constant. Second, when interest rates decrease, saving in banks and investing in bonds or other interest related investment vehicles become less profitable and attractive. Financial market participants switch into stock markets investment, leading to a rise in the demand for stocks. Stock prices go up accordingly. Third, companies with high debt in their balance sheets will benefit when interest rates decrease, resulting in higher net income and higher stock prices. It is also less costly for firms to borrow new loans to fuel their business growth, which will be favorable for firms' financial situation and stock value growth. Fourth, with lower interest rates, consumers are more willing to borrow to make big purchases. It would largely affect certain industries such as real estate and automobiles, generating a boost in companies' revenues and stock prices. Therefore, lower short-term interest rates generate higher stock prices, and the effect of expansionary monetary policy of reducing interest rates on stock price movements is positive.

However, there are several exceptions to the above situations, leading to a negative linkage between the expansionary monetary policy of reducing interest rate and the stock price movements. First, companies in the certain industries would suffer loss from the reduced interest rate. For example, a lower interest rate will generate a smaller
net interest margin -- the difference between the interest banks earn on lending money and the interest banks pay to the depositors -- for banks. This will cause a decrease in profits and stock prices in banking industry, resulting in a negative relationship between the expansionary monetary policy of reducing interest rate and the stock price. Second, foreign direct investments (FDI) make their decisions largely based on the interest rate of the target country. An inflow of the foreign direct investment can inject money into the stock market, and drives up the stock demand and its price in general. However, a lower interest rate is not attractive for foreign direct investments, and even causes domestic money to flow out, which is detrimental for the domestic stock market and stock prices. Third, in the portfolio theory elucidated by Cornell (1983), money balance and stocks are considered as two of many assets in the portfolio of investors. Since interest rate measures the opportunity cost of holding money balance, a change in interest rate will affect investors' decision about the proportion of money to be held in their portfolio. An increase in interest rate means the opportunity cost of holding money in the portfolio is higher, motivating investors to replace money with other investment vehicles, such as stocks. A higher demand for stocks will promote stock prices.

The above positive and negative relationship between the expansionary monetary policy of reducing interest rate and stock price movements may offset each other. The final linkage can be either positive or negative as stated above, depending on which force dominates the other. Hence, in theory, the ultimate effect of expansionary monetary policy by reducing interest rate can be ambiguous.

More surprising is that the second measure of expansionary monetary policy (increasing money supply) can also have either positive or negative impacts on stock market movements. The following reasons explain the positive effect of expansionary monetary policy of increasing money supply on stock prices. First, the main channel for the Federal Reserve to increase money supply is purchasing bonds and notes issued by government or government-sponsored enterprises through open market operations. By reducing the bond supply, the Federal Reserve drives up bond prices and drops bond yields accordingly. The low bond yields, in turn, reduce the borrowing cost of listed firms who also issue corporate bonds, and hence increase companies' earnings and stock prices,
leading to a positive relationship between money supply and stock prices. Second, a higher money supply allows banks to have more cash for loans. Consumers are easier to borrow to make big purchases, which will contribute to the rise of firms' revenue and stock prices. At the same time, the firms are easier to get access to loans, which provide the fuel for business expansion and stock price growth. Third, this mechanism is associated with the real activity hypothesis discussed by Cornell (1983). One of the Federal Reserve's responsibilities is to balance the money demand and the money supply in the economy. An increase in Federal Reserve's money supply hints at a higher money demand anticipated by the Federal Reserve, caused by higher anticipated future output. Higher anticipated future output will raise firms' future revenue and cash flows, leading to higher stock prices. Besides, higher anticipated future output can also tremendously improve investors' sentiment, which is favorable for stock price growth. Fourth, according to the quantity theory of money (Friedman 1961, 1988; Friedman and Schwartz 1963; Dhakal, Kandil, and Sharma 1993), a change in money supply unbalances the equilibrium position of money in the portfolio of investors with respect to other assets such as stocks. An increase in money supply generates an excess proportion of money in the portfolio, motivating investors to increase the holding of other assets such as stocks. A higher demand for stocks will induce higher stock prices. Therefore, changes in money supply display a positive relationship with stock price.

On the other hand, the expansionary monetary policy of a rise in money supply can also have negative impacts on stock prices, which is supported by Keynesian economists. According to them, the change in money supply only affects the stock market through altering expectations of future monetary policy. A positive shock in money supply is signaling a tightening monetary policy in the future, which will generate a pessimistic sentiment and a drop in stock market. Additionally, under the Keynesian assumption of sticky price, an increase in money supply will cause the real money balances to rise. Interest rates must drop to produce an offsetting rise in money demand to clear money market. Interest rate is also considered as the price of money in the money market. An increase in the money supply would reduce the price of money, which lowers the interest rate. Since there is a possible positive relationship between
interest rate and stock prices, which is illustrated above, the ultimate effect of an increase in money supply on stock prices is likely to be negative. Moreover, the stock market can also perceive the increase in money supply as a reinforcement signal that the economy is entering difficult times and the Federal Reserve is taking measures to help the declining market, which has a negative effect on market sentiment and stock performance. Lastly, higher money supply will create a higher expected future inflation. Since stock return is considered to be negatively inflation, which is claimed by existing studies (see Nelson, 1976; Fama and Schwert, 1977; Kaul, 1987), stock prices will reduce accordingly due to the high inflation. Given the fact that stock market is forward-looking and reflects market participants expectations about the future state of the economy and future action of monetary authority, a potential high inflation that caused by an increase in money supply is expected to trigger Federal Reserve's contractive monetary action, leading to a decrease in stock price.

Normally the impact of monetary policy takes some time to take effect due to the monetary policy transmission lag. However, it is possible that forward-looking investors, who price the stocks as the present value of future cash flow, will immediately discount the cash flows, generating a change in stock prices before the actual impact of the new monetary policy on firms' revenue take place. Due to the above reasons, the effect of expansionary monetary policy on stock movements can't be determined ahead.

### 3.3 Theoretical background of the asymmetric effects of monetary policy on stock price

The traditional theory explaining the asymmetric impact of monetary policy on stock price in different stock market conditions and different monetary policy regimes is the agency costs of financial intermediation (Bernanke and Gertler, 1989; Kiyotaki and Moore, 1997). The theory indicates that agency costs result in information asymmetry between firms and financial intermediaries. If there is information asymmetry in the financial markets, agents with information disadvantages behave as they are constraint financially. The degrees of financial constraints are different in different stock market condition and different regimes of monetary policy. Therefore, a monetary policy action can have different effects on stock returns in bull and bear market regimes, as well as in different monetary policy regimes.

## 4. Data

The overall price level of stock market is measured by the stock index. The most popular and influential stock indices in the U.S stock market nowadays are Dow Jones Industry Average, Standard \& Poor's 500, and NASDAQ Composite. Fortune (1998) shows that these stock indices display divergent movements, implying that different stock index represents different segments of the U.S. stock market and contributes different information about the stock market. Dow Jones Industry Average Index has the longest history and is the only price-weighted index. It only covers the largest 30 blue-chip stocks and all the stocks are listed in the New York Stock Exchange. S\&P500 is a value-weighted stock index, representing 500 stocks traded in New York Stock Exchange, American Stock Exchange, and NASDAQ stock market. The market value of stocks included in the S\&P500 range from large-capitalization to mid-capitalization. NASDAQ Composite covers more than 5000 stocks listed in the NASDAQ exchange. Most of these stocks are considered as technology stocks and small-capitalization stocks. As each stock index measures different stock market segments, it is reasonable to combine all three stock indices to study the overall movements of the U.S. stock market. A contribution of this paper is developing a better and broader composite measure for stock market price movements by capturing the clustering in movement of different stock exchanges and sectors. This is distinguished from Chauvet (1998/1999), which uses stock fundamentals such as price earnings ratio and dividend yield to extract stock market common factor.

Interest rate and monetary aggregate are two main measures of the Federal Reserve's monetary policy. As mentioned in the literature review, both federal funds rate and different measures of monetary aggregates have been used as the monetary policy target in the Federal Reserve's history. This paper uses the federal funds rate to represent the short-term policy interest rate. The Federal Reserve directly controls two short-term policy interest rates, which are discount rate and federal funds rate. The discount rate is the short-term interest rate the Federal Reserve charges depositary institutions for the loans borrowed directly from the Federal Reserve. The federal funds rate is the interest rate set by the Federal Reserve for depositary institutions to charge each other for the
short-term loans. As a measurement of interest rate monetary policy, the federal funds rate is more favorable than discount rate. In 2003, the Federal Reserve reformed the discount lending system, and set the discount rate 100 basis point higher than the federal funds rate to penalize the discount borrowing. Discount loan is no longer used regularly by the depository institutions during the normal time. It became the emergency loan of last resort during the financial crisis. The choice of federal funds rate is also supported by Bernanke and Kuttner (2005), who claim that changes in federal funds rate has the most immediate effect on financial markets. On the other hand, this paper chooses broader measure Divisia M4 and M2 as the representative of monetary aggregate. Divisia M4 is a broad monetary aggregate, containing negotiable money market securities, such as commercial paper, negotiable CDs, and T-bills. M4's components are modernized to be consistent with present market realities

This study doesn't distinguish between anticipated and unanticipated changes in money supply and interest rate. The proponents of efficient market hypothesis argue that all the information is already embedded in the stock price, and only the unanticipated changes in money supply and interest rate can affect the stock price. However, the conventional wisdom contends that efficient market hypothesis doesn't hold in the current stock market, and all available information is not embedded in the stock price. Therefore, anticipated changes in money supply and interest can also have an impact on stock price movement. Many previous studies show that anticipated changes in money supply and interest rate matter more than unanticipated changes. (see Maskay 2007)

The data is measured in monthly frequency and the sample period ranges from March 1971 to November 2012. The data is obtained from the websites of Federal Reserve Bank of St. Louis FRED database, Center for Financial Stability and Yahoo Finance.

## 5. Empirical Models

### 5.1 Empirical model for the identification of bull and bear markets

Burns and Mitchell (1946) proposed and Diebold and Rudehusch (1996) stressed two important features for the business cycle of economy: the comovement of the macroeconomic variables and the asymmetry between expansions and recessions. This is
also the principle that the National Bureau of Economic Research (NBER) uses to provide the official periods of business cycle and the dates at which the shift of economic phase take place in the United States. In order to date an economic peak, which is the turning points of the transition from an expansion to a recession, the National Bureau of Economic Research seeks for the comovement in the switch of several major economic variables from the upward growth into the decline. The economic trough, which is the turning point of the transition from an expansion phase to a recession phase, is dated by the National Bureau of Economic Research using the reversed method. The dates of business cycle turning points and its calculation method are widely accepted by the public. These two features - comovement and asymmetry - apply to the fluctuation cycle of stock market as well. First, there exists a comovement of stock prices among different sectors and different exchanges. The common dynamics of different stock prices can be represented by an unobserved common factor in a dynamic factor model, which reflects the overall movement of the stock market. The dynamic factor model, developed by Geweke (1977) Sargent and Sims (1977), and Stock and Watson (1989, 1991), successfully captures the common underlying source which generates comovements among different variables. The second feature demonstrates that stock market behaves differently during bull market regime versus bear market regime. It is possible that the growth rate or volatility is completely different in different regimes. However, a linear model is not capable to capture this asymmetry in the stock market price dynamics. Hamilton's (1989) state-dependent Markov switching model is designed to characterize this nonlinearity feature as it allows for switching of regimes.

Therefore, in order to apply the NBER's principle to date the turning points of stock market regimes and study the two features inherent in the stock market, which are comovement and asymmetry, the dynamic factor model and the state-dependent Markov-switching model become the natural choice for my research. More specifically, one aim of this paper is to combine the dynamic factor model and the state-dependent Markov switching model, and construct a new composite stock market indicator to better represent the overall movements of the U.S. stock market. The Markov-switching dynamic factor model is undertaken in the framework of a state space model, and
estimated via Kalman Filter (1960) and Hamilton Filter (1989). The dynamic factor model captures the clustering of shifts of a variety of popular stock indexes between their upward tendency and downward tendency. The Markov-switching feature reflects the asymmetry of stock movements in terms of growth rate and volatility, and is able to statistically identify the date of turning points using the transition probabilities.

Diebold and Rudebusch (1996) propose a Markov-switching dynamic factor model which encompasses these two features in one model for the first time. However, they did not actually carry out the estimation due to the heavy computational burden. Kim (1994), Kim and Yoo (1995), and Chauvet (1998) developed the Markov-switching dynamic factor model and actually undertake the estimation by maximum likelihood method to estimate both the dynamic common factor and the regime-switching transition probabilities simultaneously. This paper follows Chauvet (1998) to assume that the intercept and variance of common factor is Markov switching between different regimes. Kim and Nelson (1999) provide a detailed summary and overview, and this paper use this book as the main reference.

Markov-switching dynamic factor model is carried out within the state-space models. State-space model was originally developed by Kalman (1960), and was applied to solve dynamic problems that involve unobserved state variables. The unobserved dynamic common factor is just one component of the unobserved state vector. State-space models are made up of two equations, which are measurement equation and transition equation. Measurement equation describes the relationship between observed variables and unobserved state variables. Transition equation describes the dynamic relationship between the unobserved state variable and its own lagged terms.

The essence of a Markov-switching dynamic factor model is that one unobserved dynamic factor, $f_{t}$, captures the comovements of a vector of time-series observed variables, $Y_{t}$, which have higher dimension. The unobserved dynamic factor, which follows an autoregression, has the mean and conditional volatility that are functions of a Markov state variable $S_{t}$, with the purpose of measuring the potential asymmetries across different stock market regimes in terms of growth rate and volatility. The random variable $S_{t}$ takes the value of zero or one, and represents the regime of stock market,
either bear or bull. The vector of time-series observed variables is also impacted by a vector of idiosyncratic disturbances, $e_{t}$. These idiosyncratic disturbances capture the special features that are specific to an individual observed variable. The latent factors also follow a time series process.

In equations, the Markov-Switching dynamic factor model is presented as following,

$$
\begin{aligned}
\Delta Y_{t} & =\gamma \Delta f_{t}+\Delta e_{t} & & \\
\Delta f_{t} & =\mu_{S_{t}}+\phi \Delta f_{t-1}+w_{t}, & & w_{t} \sim \text { i.i.d. } N\left(0, \sigma_{w-S_{t}}^{2}\right) \\
e_{t} & =\varphi(L) e_{t-1}+\epsilon_{t}, & & \epsilon_{t} \sim i . i . d . N(0, \Omega) \\
\mu_{S_{t}} & =\mu_{0} S_{t}+\mu_{1}\left(1-S_{t}\right), & & S_{t}=0,1
\end{aligned}
$$

where $L$ is the lag operator and $\Delta=1-L ; \Delta f_{t}$ is a unobserved common factor extracted from the three major stock indices; $\gamma$ represents the vector of factor loadings that describes the contribution of each stock index or the sensitivity of each stock index to the common factor; $e_{t}$ denotes the vector idiosyncratic components representing the unique feature of each stock index.

In the setting of Markov switching dynamic factor model in this paper, observed time series are stock indices. This paper uses these three indices to construct the new composite measure of stock market movements. Let $Y_{t}$ be a vector of $3 \times 1$ observed variables in their $\log$ form at time $t$, which consists of Dow Jones Industry Average Index, S\&P 500 Index, and NASDAQ Index in order. Every variable can be decomposed into a common factor and a specific or idiosyncratic component. The common factor captures the simultaneous upward and downward fluctuations of stocks that are widespread in all the stock exchanges and sectors. In other words, a bear market occurs when all the three indices drop significantly at the same time and a bull market occurs when all the three indices increase simultaneously. If only one index drops, it will be captured by the idiosyncratic term of that index, rather than by a common factor.

The Markov switching from one state to another is controlled by the transition probability matrix with element $P_{i j}=p\left(S_{t}=j \mid S_{t-1}=i\right)$, where $\sum_{j=0}^{1} P_{i j}=1, i, j=0,1$. Besides, $\Delta e_{t}$ and $w_{t}$ are assumed to be mutually independent at all lags and leads. $\varphi(L)$ and $\Omega$ are diagonal based on the setting of Markov switching dynamic factor
framework. The common factor $f_{t}$ and idiosyncratic terms $e_{t}$ are assumed to be uncorrelated at all lags and leads.

The common factor and the idiosyncratic term follow a separate autoregressive process. For the dynamic factor model, it is widely accepted that the common factor follows a $\operatorname{AR}(1)$ process. However, the dynamics of the idiosyncratic terms have several possibilities. This paper estimates two most popular specifications for the idiosyncratic terms, which are $\operatorname{AR}(1)$ and $\operatorname{AR}(2)$. The first Markov-switching dynamic factor model (MSDF-Model 1) uses $\operatorname{AR}(1)$ for the idiosyncratic terms and the second Markov-switching dynamic factor model (MSDF-Model 2) uses $\operatorname{AR}(2)$ for the idiosyncratic terms.

The models are estimated by using a combination of the dynamic factor model in the state-space representation and Markov switching, as implemented by Kim (1994). In his work, he provided filtering and smoothing algorithms for the Markov-switching dynamic factor model, with a maximum likelihood estimation of unknown parameters and unobserved factors. Augmented Dickey-Fuller unit root tests are applied to each of index variable. The unit root test results show that each variable has a unit root. Johansen (1988) cointegration test is also conducted, which provides evidence that there is no cointegration relationship among these variables. According to Stock and Watson (1991), time series that have unit root but no cointegration should enter the dynamic factor model in their first differences. All the log differenced variables are then standardized by subtracting the sample mean and dividing by sample standard deviation.

The specific state-space representations for each Markov-switching dynamic factor model (MSDF-Model 1 and MSDF-Model 2) are as follows:

## MSDF-Model 1:

Measurement equation: $\Delta Y_{t}=H \beta_{t}$

$$
\left[\begin{array}{l}
\Delta Y_{1 t} \\
\Delta Y_{2 t} \\
\Delta Y_{3 t}
\end{array}\right]=\left[\begin{array}{llll}
\gamma_{1} & 1 & 0 & 0 \\
\gamma_{2} & 0 & 1 & 0 \\
\gamma_{3} & 0 & 0 & 1
\end{array}\right]\left[\begin{array}{l}
\Delta f_{t} \\
e_{1 t} \\
e_{2 t} \\
e_{3 t}
\end{array}\right]
$$

Transition equation: $\quad \beta_{t}=\mu_{S_{t}}+F \beta_{t-1}+v_{t}$

$$
\begin{aligned}
{\left[\begin{array}{l}
\Delta f_{t} \\
e_{1 t} \\
e_{2 t} \\
e_{3 t}
\end{array}\right]=} & {\left[\begin{array}{c}
\mu_{S_{t}} \\
0 \\
0 \\
0
\end{array}\right]+}
\end{aligned}+\left[\begin{array}{cccc}
\phi & 0 & 0 & 0 \\
0 & \varphi_{11} & 0 & 0 \\
0 & 0 & \varphi_{21} & 0 \\
0 & 0 & 0 & \varphi_{31}
\end{array}\right]\left[\begin{array}{l}
\Delta f_{t-1} \\
e_{1, t-1} \\
e_{2, t-1} \\
e_{3, t-1}
\end{array}\right]+\left[\begin{array}{l}
w_{t} \\
\epsilon_{1 t} \\
\epsilon_{2 t} \\
\epsilon_{3 t}
\end{array}\right] .
$$

## MSDF-Model 2:

Measurement equation: $\Delta Y_{t}=H \beta_{t}$

$$
\left[\begin{array}{l}
\Delta Y_{1 t} \\
\Delta Y_{2 t} \\
\Delta Y_{3 t}
\end{array}\right]=\left[\begin{array}{lllllll}
\gamma_{1} & 1 & 0 & 0 & 0 & 0 & 0 \\
\gamma_{2} & 0 & 0 & 1 & 0 & 0 & 0 \\
\gamma_{3} & 0 & 0 & 0 & 0 & 1 & 0
\end{array}\right]\left[\begin{array}{c}
\Delta f_{t} \\
e_{1 t} \\
e_{1 t-1} \\
e_{2 t} \\
e_{2 t-1} \\
e_{3 t} \\
e_{3 t-1}
\end{array}\right]
$$

Transition equation: $\quad \beta_{t}=\mu_{S_{t}}+F \beta_{t-1}+v_{t}$

$$
\begin{gathered}
{\left[\begin{array}{c}
\Delta f_{t} \\
e_{1 t} \\
e_{1 t-1} \\
e_{2 t} \\
e_{2 t-1} \\
e_{3 t} \\
e_{3 t-1}
\end{array}\right]=\left[\begin{array}{c}
\mu_{S_{t}} \\
0 \\
0 \\
0 \\
0 \\
0 \\
0
\end{array}\right]+\left[\begin{array}{ccccccc}
\phi & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & \varphi_{11} & \varphi_{12} & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & \varphi_{21} & \varphi_{22} & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & \varphi_{31} & \varphi_{32} \\
0 & 0 & 0 & 0 & 0 & 1 & 0
\end{array}\right]\left[\begin{array}{c}
\Delta f_{t-1} \\
e_{1 t-1} \\
e_{1 t-2} \\
e_{2 t-1} \\
e_{2 t-2} \\
e_{3 t-1} \\
e_{3 t-2}
\end{array}\right]+\left[\begin{array}{c}
w_{t} \\
\epsilon_{1 t} \\
0 \\
\epsilon_{2 t} \\
0 \\
\epsilon_{3 t} \\
0
\end{array}\right]} \\
v_{t} \sim \text { i.i.d. } N(0, Q) \\
Q=\left[\begin{array}{ccccccc}
\sigma_{w_{-} S_{t}}^{2} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & \sigma_{1}^{2} & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & \sigma_{2}^{2} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & \sigma_{3}^{2} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0
\end{array}\right]
\end{gathered}
$$

For identification, it is necessary to standardize one of the factor loadings $\gamma_{i}$ or factor variance $\sigma_{w_{-} S_{t}}^{2}$ to be one. In our model setting, the factor variance follows a Markov-switching process to capture the asymmetry between bull and bear market in terms of growth rate and volatility. Therefore, we set second factor loading $\gamma_{2}$ to one. The econometric estimation procedures are shown in the Appendix, which includes Kalman filter, Hamilton filter, smoothing, and approximations.

It is likely that the effects of monetary policy on stock performance can be
different in bear market and bull market, which is the focus of this study. This paper provides the dates of each bear market and bull market to assist the analysis of effects of monetary policy on stock performance. In order to define the turning point of bear market and bull market, we need to define the procedure for identify these turns. The above Markov-switching dynamic factor model provides probabilities that can be used as the rule. During periods classified as good stock performance, smoothed probability of bear market regime $\operatorname{pr}\left(S_{t}=0 \mid I_{T}\right)$ is mostly close to 0 . This probability spikes upward sharply and remains high when stock market enters into a bear market. Although visual inspection is helpful to measure the time periods of bear markets and bull markets, a formal definition is needed to precisely date the turning points using probabilities. The commonly accepted method used by Hamilton (1989) and Chauvet and Piger (2003), a turning point is defined to take place when smoothed probability of bear market regime $\operatorname{pr}\left(S_{t}=0 \mid I_{T}\right)$ moves across the 50 percent line, which separates the time periods when bear market is more likely from the time periods when bull markets is more likely. Therefore, the beginning date of the bear market is defined as the time point when smoothed probability of bear market regime $\operatorname{pr}\left(S_{t}=0 \mid I_{T}\right)$ changes from below 50 percent into above 50 percent. The ending date of the bear market is similarly defined as the time point when smoothed probability of bear market regime $\operatorname{pr}\left(S_{t}=0 \mid I_{T}\right)$ changes from above 50 percent into below 50 percent.

### 5.2 Empirical Model for the analysis of monetary policy's impact on stock market

The Markov-switching dynamic factor model also produces a composite index to represent the overall stock market price movements, and calculates the probability of bear market and bull market. Then this paper applies this stock price movement index into two time-varying parameter models to study the predictive and contemporaneous effect of monetary policy on stock market performance. Time-varying parameter model is chosen to study the effect of monetary policy on stock market for the following two reasons. First, the changing coefficients statistically measure the dynamic relationship between monetary policy and stock market in different time periods, which is also the focus of this study. Second, stock price reflect market participants' expectation of the future. Investors in the stock market revise their expectations when new information
becomes available. The changing coefficients capture the expectation revision of investors and show how investors have been changing the view on stock market. Second, time-varying parameter model is undertaken within the environment of a state-space model, which is calculated through a Kalman filter and the maximum likelihood estimation. As Harrison and Stevens (1976) and Kim and Nelson (1999) argue, an investor's uncertainty about the future arises not only because of the uncertainty about future random disturbance, but also from the uncertainty about the accuracy of estimated parameter values of the model. The equation in the Kalman filter for the variance of forecast error fully captures this property. In equations, the specification of the time-varying parameter model is presented as following.

## Time-Varying Parameter Model :

$$
\Delta f_{t}=\beta_{0 t}+\beta_{1 t} \Delta M 4_{t}+\beta_{2 t} \Delta i_{t}+u_{t}
$$

$\beta_{i t}=\beta_{i t-1}+\varepsilon_{i t} i=0,1,2$
Measurement equation: $\Delta f_{t}=x_{t} \beta_{t}+u_{t}$

$$
\Delta f_{t}=\left[\begin{array}{lll}
I & \Delta M 4_{t} & \Delta i_{t}
\end{array}\right]\left[\begin{array}{l}
\beta_{0 t} \\
\beta_{1 t} \\
\beta_{2 t}
\end{array}\right]+u_{t}
$$

Transition equation: $\beta_{t}=\beta_{t-1}+\varepsilon_{t}$

$$
\begin{gathered}
{\left[\begin{array}{l}
\beta_{0 t} \\
\beta_{1 t} \\
\beta_{2 t}
\end{array}\right]=\left[\begin{array}{l}
\beta_{0, t-1} \\
\beta_{1, t-1} \\
\beta_{2, t-1}
\end{array}\right]+\left[\begin{array}{l}
\varepsilon_{0 t} \\
\varepsilon_{1 t} \\
\varepsilon_{2 t}
\end{array}\right]} \\
u_{t} \sim \text { i.i.d. } N\left(0, \sigma_{u}{ }^{2}\right) \\
\varepsilon_{t} \sim \text { i.i.d. } N(0, Q) \\
Q=\left[\begin{array}{ccc}
\sigma_{0}{ }^{2} & 0 & 0 \\
0 & \sigma_{1}{ }^{2} & 0 \\
0 & 0 & \sigma_{2}{ }^{2}
\end{array}\right]
\end{gathered}
$$

where $\Delta f_{t}$ is a unobserved common factor extracted from the three major stock indices in the previous dynamic factor model measuring the overall stock price movement; $\beta_{i t}$ is time-varying coefficient which measures the relationship between monetary policy and stock prices; $\Delta M_{t}$ is the difference of log broad monetary aggregate Divisia M4; $\Delta i_{t}$ is the difference of $\log$ federal funds rate; $u_{t}$ is the error term of the time-varying regression.

The above model explores the contemporary relationship among M4, federal funds
rate and stock market. This study also investigates lead-lag relationship among M4, federal funds rate and stock market in the time-varying parameter Model 2. As shown by Friedman (1988), monetary aggregate has different contemporary relationship and leading relationship with stock prices. Considering the fact that this paper uses monthly data and many studies documented that the effects of monetary policy action on stocks are immediate, the analysis on the relationship between monetary policy and stock return with one month lag is conducted. In the time-varying parameter model 3 and 4, this paper uses a narrower money supply measurement M2 to replace M4 (see footnote ${ }^{1}$ ) for robustness check.

## 6. Empirical results

The Maximum likelihood estimation results for the parameters of Markov-switching dynamic factor models are shown in the Table 1, with standard errors in the parentheses. The estimation results of Markov-switching dynamic factor model 2 is more favorable than model 1. MSDF-Model 1 has an insignificant variance for the second idiosyncratic term $\sigma_{2}$, indicating that the common factor was dominated by the second variable S\&P500 index and the contribution of the other two indices is trivial. Besides, MSDF-Model 2 has higher $\log$ likelihood than MSDF-Model 1. Therefore, this paper adopts MSDF-Model 2 as the Markov-switching dynamic factor model.

The factor loading measures the sensitivity of each stock index to the dynamic common factor. The estimates of factor loadings $\gamma_{i}$ in the MSDF-Model 2 are all significantly positive, which means all the indices have positive contributions to the underlying common factor. The model allows the intercept and the variance of the common factor to follow Markov switching between two regimes, and they are all statistically significant and very different from its own counterpart. The intercept of bear market regime $\mu_{0}$ has expected negative sign while the intercept of bull market regime

[^0]$\mu_{1}$ has expected positive sign, implying that the underlying common factor has downward movements in bear markets but upward movements in bull markets. It is also shown by the estimation results that stock market is more volatile in bear market than bull market, given that $\sigma_{w_{-} 1}$ is larger than $\sigma_{w_{-} 2}$. Moreover, the probability for the bear market to stay in the bear market is $P_{00}=p\left(S_{t}=0 \mid S_{t-1}=0\right)=82.96 \%$. This shows that the expected duration of bear market is 5.6 months, which is calculated by using formula $1 /\left(1-P_{00}\right)$. Similarly, the probability for the bull market to stay in the bull market is $P_{11}=p\left(S_{t}=1 \mid S_{t-1}=1\right)=92.4 \%$. The expected duration of bull market is about 13.2 months, calculated by $1 /\left(1-P_{11}\right)$.

Figure 1 plots the smoothed probability of the bear market in the Markov-switching dynamic factor model. The reason for presenting smoothed probability rather than filter probability lies in the fact that the filter is based on information available up to time $t$, but the smoothing is based on all the information through time $T$. Hence smoothing has more information available than filter and hence provides a more accurate inference on the unobserved state vector and its covariance matrix.

Figure 1 successfully captures all the bear markets in the sample period, namely stock crash in 1973 mainly caused by the economy stagflation and oil price rise, 1980 Silver Thursday sharp stock price drop caused by the silver market crash, 1982 stock price huge decline impacted by Kuwait's stock market losses, 1987 Black Monday stock crash, early 1990s' stock crash caused by the burst of Japanese property price bubble, bear market in 1998 caused by Russian financial crisis, stock crash in late 2001 caused by September 11 terrorist attack, bear market in 2002 generated by the burst of internet technology bubble, stock market crash in 2007 affected by subprime mortgage crisis, and stock market downturn in 2010 and 2011 caused by European sovereign debt crisis. This provides the evidence showing that the two-state Markov switching model successfully captures the dynamics of regime changes between bear market and bull market of the U.S. stock market. This paper applies the 0.5 cut off line to the smoothed probabilities of bear market as the rule to determine the dates of bear market. The beginning and ending dates of each bear market is shown in Table 2 and the time periods of bear market is
demonstrated by the green area in Figure 2. The areas between red lines in Figure 2 denote the periods of economic recession of the U.S., which is officially announced by National Bureau of Economic Research. Figure 2 shows that every domestic economic recession is associated with a bear market, but a bear market is not necessarily associated with a domestic economic recession. It confirms one commonly accepted notion that stock market is related to the domestic economy but more volatile than the domestic economy. This is because the underlying domestic economic condition is just one of the fundamental driving factors of stock market fluctuation. However, the movements of stock market are affected by many other factors besides the underlying domestic economic condition. For instance, the fluctuations of global market influence the U.S. stock market to a large extent. The huge negative impact of European debt crisis in early 2010s is a good example, which forced the U.S. stock market to fall into a bear market, while the domestic economy stayed out of a recession. What's more, the U.S. stock market is also substantially affected by political issue, unexpected events, natural disaster, investors' fears, and etc. Most of them do not give rise to turns in business cycle of economy. Another important phenomenon demonstrated by the plot is that the stock market occasionally falls into a bear market in advance of the economic recession, confirming that stock market is a leading indicator of the economy. For example, the stock market switches into a bear market four months before the arrival of 2007 economic recession. This coincides with the many existing studies showing that the stock market index is the leading indicator of economic business cycle (see for example Chauvet 1998/1999).

Having demonstrated the time periods of U.S. bear/bull market and the features of stock market movements above, we now turn to the more difficult question of monetary policy's effects on theses stock market movements across the bull and bear market, as well as different regimes of monetary policy. Time-varying parameter model are chose to examine the potential asymmetry over time. The Maximum likelihood estimation results for time-varying parameter models to study lead-lag relationship and concurrent relationship are shown through Table 3 and Table 6, with standard errors in parentheses.

Figure 3 plots the time-varying coefficients $\beta_{1 t}$ which measures the contemporary
relationship between broad monetary aggregate Divisia M4 and stock movements. It is shown that there is a sharp drop in the time-varying parameter in every bear market, indicating the expansionary monetary policy of increasing monetary aggregate is less effective during a bear market. The sign of time-varying parameter has switched from positive to negative since 1987. 1987 is the year when Alan Greenspan became the Federal Reserve chairman and abandoned the monetary aggregate as the monetary target. This leads the conclusion that the signaling effect of monetary policy action of changing monetary aggregate only functions during the periods when it is used as the monetary policy target. A further interpretation of this result is that the Federal Reserve's action of changing monetary aggregate has positive effects on stock return only if it is considered by the market participants as a meaningful indicator of monetary policy. If the monetary aggregate is not used monetary target, the stock market may not respond to the changes in monetary aggregate in a regular manner, and the negative impacts of monetary aggregate increase on stock performance that explained in the theoretical background would dominate the positive effects. During a bear market, a drop in the correlation makes the negative relationship more negative, which arrives at the conclusion that an expansionary monetary policy action of increasing monetary aggregate can even deteriorate the stock performance during a bear market within the periods when monetary aggregate is not the policy target.

As is evident from Figure 4, the concurrent relationship between changes in federal funds rate and stock price movements is inconsistent, switching between positive and negative as expected. The positive coefficient means the positive effects shown in the previous theoretical framework section dominate the negative effects, and vice versa. During the periods (1974-1980, and 1990-2008) that the federal funds rate was used as a monetary policy target, the sign of the relationship between federal funds rate and stock market is negative, indicating that the expansionary monetary policy of reducing federal funds rate is effective in improve stock performance. This parameter becomes positive during other periods (1980s and after 2008), which illustrates that monetary action of reducing federal funds rate is useless in improving stock performance. This dynamics reinforces the conclusion that the signaling effects of monetary policy influence investors'
sentiment successfully only when the market participants believe the Federal Reserve's action is meaningful. This result is supported by Clarida et al. (2000) who found that the federal funds rate has volatile effects on stock market. Besides, the coefficient also has a sharp decrease during every bear market. These drops make a positive coefficient negative, and a negative coefficient even more negative. If the government would like to apply an expansionary monetary policy to stimulate the stock market by reducing the federal funds rate in a bear market, it will be effective, given that it is during the periods when federal funds rate is used as an effective monetary policy target. This result is consistent with the findings of Jansen and Tsai (2010) and Kurov (2010).

Several distinct results emerge from Figure 5, which plots the time-varying coefficients $\beta_{1 t}$ which measures the predictive relationship between broad monetary aggregate Divisia M4 and stock price one month later. One result refers to the fact that there exists a sharp drop in the value of the coefficient in every bear market, indicating that the leading effect of monetary policy of changing monetary aggregate is much weaker in a bear market. In most bear markets, the coefficient reduces even below zero, presenting a negative relationship between money supply and stock market. If the Federal Reserve uses expansionary monetary policy to improve stock market performance during a bear market by increasing money supply, it is very ineffective and may even deteriorate the stock market. It illustrates that money supply is positively associated with future stock performance during most bull markets, with the exception of time periods in early 1990s and 2000s. In fact, the most recent two economic recessions in 2000s were all followed by a slow and sluggish economy recovery. The economic recession in early 1990s was also followed by a four-year slow recovery, and the economy started to take off in the middle of 1990s. This arrives at a conclusion that a positive predictive relationship between money supply and stock market occurs during the periods of robust economic growth, but not during the periods of economic recession or slow recovery. The lead-lag relationship between monetary policy and stock market is more related to the business cycle than the regimes of monetary policy.

Figure 6 depicts the dynamic association between the changes in stock prices and changes in federal funds rate. It shows the predictive relationship between changes in
federal funds rate and stock price movements is negative during all periods. This finding provides the evidence that the expansionary monetary policy of reducing federal funds rate is very effective in all regimes of monetary policy and all regimes of stock market. This negative relationship becomes weaker since late 2008, where the coefficient of lagged federal funds rate is close to zero. This is due to the fact that the level of the federal funds rate was reduced to the zero lower bound in late 2008, and can't be used as an expansionary monetary tool for further reduction.

If we replace M4 with M2 in time-varying parameter model 3 and 4, the results are similar. The dynamic pattern of federal funds rate is the same as in model 1 and 2 (see Figure 8 and 10). Figure 7 shows that the concurrent relationship between M2 and stock market is similar to that between M4 and stock. However, the lead-lag relationship between M2 and stock market (see Figure 9) is strikingly different from that between M4 and stock. The curve is very flat and the insignificant parameter of variance indicates that there is no too much volatility in the relationship. The relationship remains positive until 1987, where the parameter reduces fundamentally to zero. This is consistent with the previous finding that the monetary aggregate change's signaling effect only works during periods when monetary aggregate is used as the monetary policy target. The relationship turns into negative during the 2007 financial crisis. The lead-lag relationship between M2 and stock performance does not demonstrate a distinguished feature in different regimes of stock market and different phases of business cycle, confirming that M4 is a broader measure of monetary aggregate.

## 7. Conclusion

As mentioned in the introduction, previous literatures found that the Federal Reserve's monetary policy has played an important role in affecting stock returns, but the empirical literature on the asymmetric effects of monetary policy on stock returns over time is limited and, unfortunately, mixed. The purpose of this paper is to improve on the earlier literature by conducting another empirical analysis of the time-varying effects of monetary policy on stock performance in different monetary policy regimes and stock market regimes during the last four decades. More specifically, how have the different
views on applying monetary policy by Burns in the 1970s, Volcker in the 1980s, Greenspan in the 1990s and early 2000s, and Bernanke from mid 2000s to present affected the stock market? How has the nature of the dynamic relationship between monetary policy and stock return vary during the bull and bear market? The substantial stock market volatility under current expansionary monetary policy emphasizes the necessity and urgency of the study on this issue.

This paper begins with the exploration of the dates of the turning points of bear and bull market by applying a Markov-switching dynamic factor model on major stock indices, and produces a new composite measure to represent the overall stock market movement more broadly and comprehensively. The Markov-switching dynamic factor model extracts the comovement among stocks across different sectors and stock exchanges with an unobserved underlying common factor. The Markov-switching feature catches the nonlinear asymmetry in bear and bull market in terms of growth rate and volatility because of its nonlinearity setting, and is capable of statistically identifying the turning points of stock market regimes by using its inherent transition probabilities. It estimates the probabilities of bear market and bull market of every time point in the sample periods. The results successfully capture all the bear markets in the sample history. The findings indicate bear markets are more volatile than bull markets, and the average durations of bear market is shorter than that of bull market. The paper shows that bear markets frequently occur in advance of economic recessions, confirming that stock market is a leading indicator of business cycle of economy. It is also shown that every domestic economic recession is associated with a bear market, but not vice versa. This coincides with the widely accepted notion that underlying domestic economic condition is the most essential driving force for stock market fluctuation, but the stock market fluctuation is also affected by many other factors and information as well. These findings can be instrumental in helping investors to understand in which state of the fluctuation cycle the stock market is and where the stock market is moving towards.

Having illustrated the characteristics of U.S. stock market movements above, this paper turns to the more difficult question of the dynamic relationship between these stock market movements and monetary policy. The newly extracted unobserved factor is
then applied into a time-varying parameter model as a composite measure of stock market movements. The results provide the evidence that the relationship between monetary policy and stock returns varies over time, and the responses of stock returns to monetary policy are asymmetric during bull and bear markets, and across different monetary policy regimes. Specifically, the contemporary signaling effects of increases in monetary aggregates or reductions in federal funds rate are positive on stock returns only during periods when they are used as the monetary policy target by the Federal Reserve. In other words, Federal Reserve's action of changing monetary aggregates or federal funds rate is effective on stock market only if it is considered by the market participants as a meaningful indicator of monetary policy. Besides, a positive predictive relationship between monetary aggregate and stock returns one month later is detected during the periods of robust economic growth, but not during the periods of economic recession or slow recovery. The observation of a sharp drop in the value of the correlation between monetary aggregate and stock return in every bear market indicates that the impacts of the monetary policy of increasing monetary aggregates are much weaker in a bear market, and can even deteriorate stock market. However, the expansionary monetary policy of reducing federal funds rate is effective in improving stock market performance during a bear market within the periods when federal funds rate is used as intermediate target by the Federal Reserve.

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## Appendix:

## Estimation procedure of Markov-switching dynamic factor model

Let $I_{t}$ denote the information set which contains the observations available up to time $t$. In Markov-switching dynamic factor model, the forecast of unobserved state vector $\beta_{t}$ is not only dependent on information set $I_{t-1}$, but also based on state variable $S_{t}$ that takes on the value of $j$ and $S_{t-1}$ that takes on the value of $i$. The forecast of state variable $\beta_{t}$ and its covariance matrix is as follows:

$$
\begin{gathered}
\beta_{t \mid t-1}^{(i, j)}=E\left[\beta_{t} \mid I_{t-1}, S_{t-1}=i, S_{t}=j\right] \\
P_{t \mid t-1}^{(i, j)}=E\left[\left(\beta_{t}-\beta_{t \mid t-1}\right)\left(\left(\beta_{t}-\beta_{t \mid t-1}\right)^{\prime} \mid I_{t-1}, S_{t-1}=i, S_{t}=j\right]\right.
\end{gathered}
$$

Based on Markov switching states $S_{t-1}=i$ and $S_{t}=j$, the Kalman filter is:

$$
\begin{gathered}
\beta_{t \mid t-1}^{(i, j)}=\mu_{j}+F_{j} \beta_{t-1 \mid t-1}^{(i)} \\
P_{t \mid t-1}^{(i, j)}=F_{j} P_{t-1 \mid t-1}^{(i)} F_{j}^{\prime}+Q_{j} \\
\theta_{t \mid t-1}^{(i, j)}=\Delta Y_{t}-\Delta Y_{t \mid t-1}^{(i, j)}=\Delta Y_{t}-H_{j} \beta_{t \mid t-1}^{(i, j)} \\
\tau_{t \mid t-1}^{(i, j)}=H_{j} P_{t \mid t-1}^{(i, j)} H_{j}^{\prime} \\
\beta_{t \mid t}^{(i, j)}=\beta_{t \mid t-1}^{(i, j)}+P_{t \mid t-1}^{(i, j)} H_{j}^{\prime}\left[\eta_{t \mid t-1}^{(i, j)}\right]^{-1} \theta_{t \mid t-1}^{(i, j)}=\beta_{t \mid t-1}^{(i, j)}+K_{t} \theta_{t \mid t-1}^{(i, j)} \\
P_{t \mid t}^{(i, j)}=\left(I-P_{t \mid t-1}^{(i, j)} H_{j}^{\prime}\left[\tau_{t \mid t-1}^{(i, j)}\right]^{-1} H_{j}\right) P_{t \mid t-1}^{(i, j)}
\end{gathered}
$$

where $\beta_{t-1 \mid t-1}^{(i)}$ and $P_{t-1 \mid t-1}^{(i)}$ are inferences on $\beta_{t-1}$ and $P_{t-1}$ conditional on information up to time $t-1$ and $S_{t-1}=i ; \theta_{t \mid t-1}^{(i, j)}$ is the prediction error of $y_{t}$ conditional on information up to time $t-1$, given the values of the two states $S_{t-1}=i$ and $S_{t}=j$; and $\tau_{t \mid t-1}^{(i, j)}$ is the conditional variance of the prediction error. The details of the derivation of the above Kalman filter can be refereed to Hamilton (1994).

In order to make the loop of above Kalman filter operable, it is necessary to transfer $\beta_{t \mid t}^{(i, j)}$ and $P_{t \mid t}^{(i, j)}$ at the end of the each iteration into $\beta_{t \mid t}^{(j)}$ and $P_{t \mid t}^{(j)}$, and use $\beta_{t \mid t}^{(j)}$ and $P_{t \mid t}^{(j)}$ to represent $\beta_{t-1 \mid t-1}^{(i)}$ and $P_{t-1 \mid t-1}^{(i)}$ for the next period. Kim (1994) proposed an
algorithm for the transfer. This algorithm involves approximation, and is proved to be accurate. The procedure is as follows:

$$
\begin{gathered}
\beta_{t \mid t}^{(j)}=\left[\sum p r\left(S_{t-1}=i, S_{t}=j \mid I_{t}\right) \beta_{t \mid t}^{(i, j)}\right] / \operatorname{pr}\left(S_{t}=j \mid I_{t}\right) \\
P_{t \mid t}^{(j)}=\left[\sum p r\left(S_{t-1}=i, S_{t}=j \mid I_{t}\right) P_{t \mid t}^{(i, j)}+\left(\beta_{t \mid t}^{(j)}-\beta_{t \mid t}^{(i, j)}\right)\left(\beta_{t \mid t}^{(j)}-\beta_{t \mid t}^{(i, j)}\right)\right] / \operatorname{pr}\left(S_{t}=j \mid I_{t}\right)
\end{gathered}
$$

The probability terms $p\left(S_{t-1}=i, S_{t}=j \mid I_{t}\right)$ and $\operatorname{pr}\left(S_{t}=j \mid I_{t}\right)$ in the above equations have to be estimated to complete the Kalman filter involving approximation. By using Hamilton (1989) filter along with Markov switching, the inference on the above probability terms can be calculated and shown as follows:

$$
\begin{gathered}
p\left(S_{t-1}=i, S_{t}=j \mid I_{t-1}\right)=\operatorname{pr}\left(S_{t-1}=i \mid I_{t-1}\right) \operatorname{pr}\left(S_{t}=j \mid S_{t-1}=i\right) \\
f\left(y_{t}, S_{t-1}=i, S_{t}=j \mid I_{t-1}\right)=f\left(y_{t} \mid S_{t-1}=i, S_{t}=j, I_{t-1}\right) \operatorname{pr}\left(S_{t-1}=i, S_{t}=j \mid I_{t-1}\right) \\
f\left(y_{t} \mid I_{t-1}\right)=\sum_{S_{t-1}=i} \sum_{S_{t}=j} f\left(y_{t}, S_{t-1}=i, S_{t}=j \mid I_{t-1}\right) \\
\operatorname{pr}\left(S_{t-1}=i, S_{t}=j \mid I_{t}\right)=\operatorname{pr}\left(S_{t-1}=i, S_{t}=j \mid y_{t}, I_{t-1}\right)=f\left(y_{t}, S_{t-1}=i, S_{t}=j \mid I_{t-1}\right) / f\left(y_{t} \mid I_{t-1}\right) \\
=f\left(y_{t} \mid S_{t-1}=i, S_{t}=j, I_{t-1}\right) \operatorname{pr}\left(S_{t-1}=i, S_{t}=j \mid I_{t-1}\right) / f\left(y_{t} \mid I_{t-1}\right) \\
\operatorname{pr}\left(S_{t}=j \mid I_{t}\right)=\sum_{i} \operatorname{pr}\left(S_{t-1}=i, S_{t}=j \mid I_{t}\right)
\end{gathered}
$$

The transition probabilities capture the Markov switching between two states and are estimated by Maximum Likelihood estimation as one of the unknown parameters. For the inference of conditional density $f\left(y_{t} \mid S_{t-1}=i, S_{t}=j, I_{t-1}\right)$, prediction error decomposition involving conditional forecast error and its variance obtained from the previous Kalman filter is used as follows.

$$
\begin{gathered}
f\left(y_{t} \mid S_{t-1}=i, S_{t}=j, I_{t-1}\right)=(2 \pi)^{-N / 2}\left|\left[\eta_{t \mid t-1}^{(i, j)}\right]^{-1 / 2}\right| \exp \left\{-\frac{1}{2} P_{t \mid t-1}^{(i, j)} H_{j}^{\prime}\left[\eta_{t \mid t-1}^{(i, j)}\right]^{-1} \theta_{t \mid t-1}^{(i, j)}\right\} \\
f\left(y_{t} \mid I_{t-1}\right)=\sum_{S_{t}} \Sigma_{S_{t-1}} f\left(y_{t} \mid S_{t-1}=i, S_{t}=j, I_{t-1}\right) p\left(S_{t-1}=i, S_{t}=j \mid I_{t-1}\right) \\
l(\theta)=\sum_{t=1}^{\mathrm{T}} \ln \left(f\left(y_{t} \mid I_{t-1}\right)\right)
\end{gathered}
$$

Initial values $\beta_{0 \mid 0}^{(j)}$ and $P_{0 \mid 0}^{(j)}$ for Kalman filter and $\operatorname{pr}\left(S_{0}=j \mid I_{0}\right)$ for Hamilton filter are assigned to start the iteration. After the Kalman filter and Hamilton filter are completed, smoothing procedures for $\beta_{t}, P_{t}$ and probability terms begin. The smoothing algorithm iterates backwards and has the following procedure:

$$
\beta_{t \mid T}^{(j, k)}=\beta_{t \mid t}^{(j)}+P_{t \mid t}^{(j)} F_{k}^{\prime}\left[P_{t+1 \mid t}^{(j, k)}\right]^{-1}\left(\beta_{t+1 \mid T}^{(k)}-\beta_{t+1 \mid t}^{(j, k)}\right)
$$

$$
\begin{gathered}
P_{t \mid T}^{(j, k)}=P_{t \mid t}^{(j)}+P_{t \mid t}^{(j)} F_{k}^{\prime}\left[P_{t+1 \mid t}^{(j, k)}\right]^{-1}\left(P_{t+1 \mid T}^{(k)}-P_{t+1 \mid t}^{(j, k)}\right) P_{t \mid t}^{(j)} F_{k}^{\prime}\left[P_{t+1 \mid t}^{(j, k)}\right]^{-1}, \\
\operatorname{pr}\left(S_{t}=j, S_{t+1}=k \mid \varphi_{T}\right) \approx \operatorname{pr}\left(\mathrm{S}_{\mathrm{t}+1}=\mathrm{k} \mid \phi_{\mathrm{T}}\right) \operatorname{pr}\left(\mathrm{S}_{\mathrm{t}}=\mathrm{j} \mid \phi_{\mathrm{t}}\right) \operatorname{pr}\left(\mathrm{S}_{\mathrm{t}+1}=\mathrm{k} \mid \mathrm{S}_{\mathrm{t}}=\mathrm{j}\right) / \operatorname{pr}\left(\mathrm{S}_{\mathrm{t}+1}=\mathrm{k} \mid \phi_{\mathrm{t}}\right) \\
\operatorname{pr}\left(S_{t}=j \mid \varphi_{T}\right)=\sum_{k=0}^{1} \operatorname{pr}\left(S_{t}=j, S_{t+1}=k \mid \varphi_{T}\right) r
\end{gathered}
$$

The initial values for the smoothing $\beta_{\mathrm{T} \mid \mathrm{T}^{\prime}}^{(\mathrm{k}}, \mathrm{P}_{\mathrm{T} \mid \mathrm{T}}^{(\mathrm{k})}$ are obtained from the last iteration of Kalman filter and Hamilton filter. The smoothing algorithm also need to transfer $\beta_{\mathrm{t} \mid \mathrm{T}}^{(\mathrm{j}, \mathrm{k})}$ and $P_{t \mid T}^{(j, k)}$ into $\beta_{t \mid T}^{(j)}$ and $P_{t \mid T}^{(j)}$. The calculation method is similar to the one with filters.

## Estimation procedure of time-varying parameter model

In the simple state space model without Markov switching, the goal of Kalman filter is to use a recursive process to produce a forecast of unobserved state vector $\beta_{t}$ and its covariance matrix with information available up to time $t-1$. They do not dependent on state information. The forecast of $\beta_{t}$ and its covariance matrix of $P_{t \mid t-1}$ are denoted as

$$
\begin{aligned}
& \beta_{t \mid t-1}=E\left[\beta_{t} \mid I_{t-1}\right] \\
& P_{t \mid t-1}=E\left[\left(\beta_{t}-\beta_{t \mid t-1}\right)\left(\left(\beta_{t}-\beta_{t \mid t-1}\right)^{\prime} \mid I_{t-1}\right] .\right.
\end{aligned}
$$

The Kalman filter iteration process is as follows:

$$
\begin{gathered}
\beta_{t \mid t-1}=\mu+F \beta_{t-1 \mid t-1} \\
P_{t \mid t-1}=F P_{t-1 \mid t-1} F^{\prime}+Q \\
\theta_{t \mid t-1}=y_{t}-x_{t} \beta_{t \mid t-1} \\
\tau_{t \mid t-1}=x_{t} P_{t \mid t-1} x_{t}^{\prime}+\sigma_{u}^{2} \\
\beta_{t \mid t}=\beta_{t \mid t-1}+P_{t \mid t-1} x_{t}^{\prime}\left[\tau_{t \mid t-1}\right]^{-1} \theta_{t \mid t-1} \\
P_{t \mid t}=\left(I-P_{t \mid t-1} x_{t}^{\prime}\left[\eta_{t \mid t-1}\right]^{-1} x_{t}\right) P_{t \mid t-1}
\end{gathered}
$$

where $\theta_{t \mid t-1}$ is the prediction error of $y_{t}$ conditional on information up to time $t-1$; and $\tau_{t \mid t-1}$ is the conditional variance of the prediction error. Initial value of $\beta_{0 \mid 0}$ and $P_{0 \mid 0}$ are given to start the Kalman filter iteration. Maximum likelihood estimation is conducted for unknown parameters based on the prediction error decomposition. The forecasting error variance equation tells that an investor's uncertainty about the future arises not only from the uncertainty about future random terms, but also from the uncertainty about the accuracy of parameter values of the model.

Table 1: The Estimation Results of Markov-Switching Dynamic Factor Models

| Parameters | MSDF-Model 1 | MSDF-Model 2 |
| :---: | :---: | :---: |
| $\phi$ | 0.2133 (0.0435) | 0.2157 (0.0435) |
| $\varphi_{11}$ | 0.2690 (0.0430) | 0.3032 (0.0456) |
| $\varphi_{12}$ |  | -0.0230 (0.0069) |
| $\varphi_{21}$ | 0.1080 (0.0000) | -0.0907 (0.0816) |
| Q 2 |  | -0.9219 (0.0512) |
| 91 | 0.3451 (0.0421) | 0.3730 (0.0454) |
| Q 2 |  | -0.0348 (0.0085) |
| P | 0.2967 (0.0094) | 0.2875 (0.0098) |
| (2) | 0.0002 (0.0069) | 0.0246 (0.0114) |
| () | 0.4532 (0.0143) | 0.4521 (0.0145) |
| Q | 1.4231 (0.1056) | 1.4158 (0.1060) |
| (8) | 0.6216 (0.0354) | 0.6156 (0.0358) |
| P | 0.9551 (0.0138) | 0.9644 (0.0141) |
| () | 0.8551 (0.0211) | 0.8590 (0.0214) |
| (9) | -0.3762 (0.1498) | -0.3826 (0.1491) |
| P | 0.1398 (0.0424) | 0.1427 (0.0426) |
| 88 | 0.8296 (0.0702) | 0.8219 (0.0747) |
| 91 | 0.9269 (0.0280) | 0.9240 (0.0298) |
| Log likelihood value | 314.6 | 321.5 |

The second and third columns are the parameters estimated via maximum likelihood estimation within the framework of two separate state space models. Standard errors are presented in the parentheses.

Table 2: The Dates of Turning Points of Bear Market

| Beginning Date (Peak) | Ending Date (Trough) |
| :---: | :---: |
| November 1971 | November 1971 |
| October 1973 | February 1975 |
| July 1975 | September 1975 |
| August 1978 | November 1978 |
| March 1980 | April 1980 |
| July 1981 | September 1982 |
| February 1984 | February 1984 |
| September 1987 | November 1987 |
| August 1998 | October 1990 |
| September 2000 | October 1998 |
| August 2007 | February 2003 |
| March 2010 | March 2009 |
| June 2011 | May 2010 |

The dates of the turning points of stock market regimes are determined using 0.5 line as the threshold
for smoothed probability of bear market for smoothed probability of bear market

Table 3: The Estimation Results of Time-Varying Parameter Model 1

| Parameters |  | Time-Varying Model |
| :--- | :--- | :--- |
|  | $\sigma_{\mathrm{u}}$ | $0.9374(0.0323)$ |
|  | $\sigma_{0}$ | $0.0001(0.0102)$ |
|  | $\sigma_{1}$ | $0.0956(0.0403)$ |
|  | $\sigma_{2}$ | $0.0013(0.0007)$ |
| Log likelihood value | 709.88 |  |

Standard errors are presented in the parentheses

Table 4: The Estimation Results of Time-Varying Parameter Model 2

| Parameters | Time-Varying Model |
| :--- | :--- |
| \& | $0.8745(0.0324)$ |
| Q | $0.0376(0.0208)$ |
| Q | $0.1259(0.0379)$ |
| Log likelihood value | $0.0068(0.0029)$ |
| Standard errors are presented in the parentheses | 697.39 |

Table 5: The Estimation Results of Time-Varying Parameter Model 3

| Parameters |  | Time-Varying Model |
| :--- | :--- | :--- |
| \& | $0.9776(0.0313)$ |  |
|  | 甲 | $0.0001(0.0162)$ |
|  | P | $0.0107(0.0190)$ |
|  | Q | $0.0015(0.0008)$ |
| Log likelihood value | 717.48 |  |

Standard errors are presented in the parentheses

Table 6: The Estimation Results of Time-Varying Parameter Model 4

| Parameters | Time-Varying Model |  |
| :--- | :--- | :--- |
| Q | $0.8779(0.0341)$ |  |
|  | Q | $0.0408(0.0195)$ |
|  | Q | $0.0839(0.0478)$ |
| Log likelihood value | $0.0122(0.0039)$ |  |

Standard errors are presented in the parentheses

Figure 1: The Smoothed Probability of Bear Market for the U.S. Stock Market


Note: the black curve plots the smoothed probabilities of bear market in each time point

Figure 2: The Periods of Bear Market and Economic Recession


The black curve plots the smoothed probabilities of bear market in each time point, green shaded areas represent the identified bear market periods, the red lines denote the beginning and ending time of NBER economic recession, and the horizontal dotted line is the zero line

Figure 3: Money Supply Parameter £jn Time-Varying Model 1


The black curve plots the time-varying parameter that measures the contemporary relationship between Divisia M4 and stock return

Figure 4: Interest Rate Parameter \&in Time-Varying Model 1


The black curve plots the time-varying parameter that measures the contemporary relationship between federal funds rate and stock return

Figure 5: Money Supply Parameter 』in Time-Varying Model 2


The black curve plots the time-varying parameter that measures the leading relationship between Divisia M4 and stock return one month later

Figure 6: Interest Rate Parameter थj Time-Varying Model 2


The black curve plots the time-varying parameter that measures the leading relationship between federal funds rate and stock return one month later

Figure 7: Money Supply Parameter £jn Time-Varying Model 3


The black curve plots the time-varying parameter that measures the contemporary relationship between M2 and stock return

Figure 8: Interest Rate Parameter ©in Time-Varying Model 3


The black curve plots the time-varying parameter that measures the contemporary relationship between federal funds rate and stock return

Figure 9: Money Supply Parameter ©in Time-Varying Model 4


The black curve plots the time-varying parameter that measures the leading relationship between M2 and stock return one month later

Figure 10: Interest Rate Parameter din Time-Varying Model 4


The black curve plots the time-varying parameter that measures the leading relationship between federal funds rate and stock return one month later


[^0]:    ${ }^{1}$ Time-Varying Parameter Model 2: $\Delta f_{t}=\beta_{0 t}+\beta_{1 t} \Delta M 4_{t-1}+\beta_{2 t} \Delta i_{t-1}+u_{t}$, Time-Varying Parameter Model 3:
    $\Delta f_{t}=\beta_{0 t}+\beta_{1 t} \Delta M 2_{t}+\beta_{2 t} \Delta i_{t}+u_{t}$,Time-Varying Parameter Model 4: $\Delta f_{t}=\beta_{0 t}+\beta_{1 t} \Delta M 2_{t-1}+\beta_{2 t} \Delta i_{t-1}+u_{t}$

